**Deriving taper function and volume ratio equation for *Shorea robusta***

Taper function and volume ratio is essential to assess NFI data as well as understanding forest growth and ecology. Estimation of tree total volume and component volumes at several tree heights has always been essential in sustainable forest management.

Sustainable forest management incorporates the use of statistical tools for estimating the total and merchantable volume of trees for the correct application of silvicultural treatments in a speciﬁc stand or group of stands, estimating the distribution of commercial products and the economic yield [1]. Forest species, site quality, stand density, and silviculturaltreatmentshaveadirectrelationshipwiththetreeproﬁle[2–4]andtheseelementsare used in the formulation of taper and volume systems [1,4,5]. In the last decades, numerous taper models have been proposed, starting with simple hyperbolic expressions, polynomial functions, segmented equations, or spline functions, and directly through volume ratio functions [6,7]. However, several studies have considered that the form of the stem does not correspond to a single equation of dendrometric type, but that the form differs to some extent with the tree species and the point in height along the stem [8]. Furthermore, the development during recent years of new ﬂexible

approachestoderivecompatibletaperandvolumesystemstowhichdatacorrespondingto different species and trees with variable stem forms can be accurately ﬁtted (e.g., [1,9–11]). Therefore, this is a good justiﬁcation for reviewing the appropriateness of compatible taper and volume equations systems derived from volume ratio functions which are based on relative tree height for different tree species. Ratio volume equations estimate the volume of a tree up to a certain diameter or height as a percentage of the total tree volume (stem volume and branch volume) [12–15]. These models consider the ﬁtting of an equation that characterizes the tree form and are mathematical relationships between the diameters or sections of the stem at any point of the tree and the height at which the diameter is estimated [16]. However, up to date most of the research related to the estimation of tree volume has focused on developing compatible tree taper and stem volume models that describe the stem proﬁle and estimate the total or partial stem volume, without considering the volume of branches [17,18]. The total tree volume models that include the branch component are requiredtoprovidebaseinformationforthesustainableuseoftheentiretreevolume[17,19] because in Mexican forestry the total tree volume is the most important variable for forest managementandplanning. Thesuitableestimationofthebranchvolumeisveryimportant because these sections are increasingly used not only for bioenergy purposes but also for quantiﬁcation of CO2 sequestration in forest ecosystems [20,21]. The estimation of branch volume is also relevant for assessing the potential bioenergy purposes on the soil nutrient and carbon cycle [22]. Theobjectiveofthisstudywastoﬁtthreecompatibletaperandmerchantableoutsidebark volume equations as simultaneous systems for estimating the taper (d, cm), merchantable outside-bark volume (Vm, m3), stem outside-bark volume (Vs, m3), branch volume (Vb, m3), and total tree volume (Vt, m3) components for main pine species in Oaxaca, Mexico. The model ﬁtting was tested for all combined species with a reduced equations system with the same set of global parameters for the four studied tree species and with a full equation system obtained by expanding in the best reduced system each global parameter by including an associated parameter and a dummy variable to differentiate the species and to assess whether separate models are necessary for the selected pine species.